TUNNELWORKS
KS4 MATHS LESSON 3 (ESSENTIALS)
TEACHERS' NOTES

About this lesson

In this lesson students explore how to model some simple real-life situations using graphs of the form $y = mx + c$. Using a simple example, students first review their understanding before finding out how barges remove spoil (waste material) from a Thames Tideway Tunnel site. They then apply their straight line graph drawing skills to compare barge designs and identify the best model to use to remove spoil from a site. Additional tasks extend these skills to compare new barge designs and consider how very low tides may limit access to a tidal site.

Learning outcomes

Students can:
- Explain what the gradient m and intercept c represent in a simple line graph of form $y = mx + c$
- Sketch a straight line graph to represent and model a simple scenario
- Interpret a straight line graph to derive a solution to a problem

Curriculum links

KS4 Maths
- Algebra – straight line graphs of the form $y = mx + c$.

What you will need

- KS4 Maths shafts lesson three presentation
- Lesson three worksheet
- Graph paper

Preparation

Review the lesson plan below and the KS4 Maths shafts lesson three presentation. Adapt the content to suit your students' ability.

Time (60mins)	Teaching activity	Learning activity	Assessment for learning
5 mins	Watch the project intro video on the Tunnelworks KS4 Maths landing page if required, and then the barges video clip on **screen 1**.	Students watch video clips.	
5 mins	Show **screen 2a**. Explain to students that this lesson is all about line graphs. Ask students to work in pairs and sketch a graph that shows the cost of hiring two barges, one at £1000 per day and one at £2000. Invite students to share their sketches. Identify that the gradient of one line should be twice that of the other. Show **screen 2b**. Now explain that there is also a fixed charge of £1500 regardless of how long the barge is hired for. Share sketches and identify the role of an intercept.	In pairs, students create a sketch graph with two lines starting at the origin. They identify that the gradient represents the day charge for each barge. Students add another line starting at **y = 1500**.	Written work, questioning, discussion.
5 mins	**Whole class:** Show **screen 3**. Review the intro paragraph on the student worksheet, then review task 1.	Students read the worksheet.	Discussion, questioning.
15 mins	**Pairs or individuals:** Review the table and ask students to accurately draw the four line graphs. They can do this as four separate graphs, or four lines on one graph. (You may wish to help students identify that the draft when empty = c.) **Whole class:** Invite students to share their graphs and identify the two barges that will float in 2.4m water.	Students complete task 1 and draw four line graphs. Students identify barges A and B as suitable, explaining their reasoning.	Written work. Discussion, questioning.
15 mins	**Whole class:** Show **screen 4**. Review task 2 and the table. **Individuals or pairs:** Students draw two further line graphs, remembering to double the time to dock / undock to find the intercept (c) value.	Students complete task 2 and draw two line graphs.	Written work, questioning, discussion.

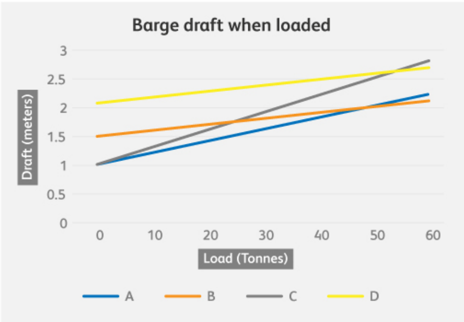
	Whole class: Invite students to share their graphs and answer.	Students identify that barge **X** is most suitable as it can be loaded more quickly.	
15 mins	**Plenary:** As time permits, explore tasks 3 and 4. Review each task with students and identify key information and steps. (For task 3 students should start at 8am and note that a barge must have left the site before 2pm. Barges can't restart loading before 4pm.)	Students complete tasks 3 and / or 4 as time permits.	Written work, questioning, discussion.

Differentiation

Easier	Harder
Task 1: Sketch the graph for barge A as a worked example for students to model. Share barges between students. Task 2: Sketch the graph for barge X as a worked example. Omit task 3. Complete task 4 as a whole-class activity on your board.	Challenge students to complete their graphs without help, identifying m and y for themselves. Students draw all graphs. Students complete tasks 3 and 4.

Answers

Answers - Task 1



Barges A and B will float in 2.4m water when loaded with 50 tonnes of spoil.

Answers - Task 2

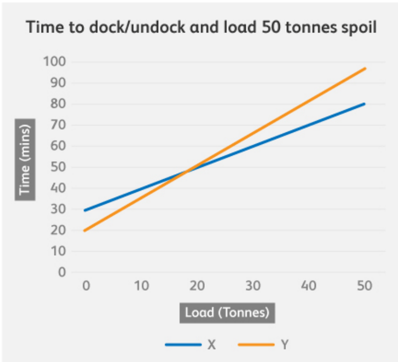
For each barge:

The constant c = total dock and undock time (30 minutes for barge X and 20 minutes for barge Y)

m = the time to load 10 tonnes (10 minutes for X and 15 minutes for Y).

X takes 80 minutes in total and Y takes 95 minutes, so **X is faster to load**.

From 8 am to 6pm is 10 hours, or 600 minutes. Barges of type X can dock, load and undock 600/80 = **7 times** during one working day (students need to round down).



Answers - Task 3

The barge can only load between 8am and 2pm, and then from 4pm to 6pm. It can't be at the site between those times.

From 8am to 2pm is 360 minutes. 360/80 = 4 times, first arriving at 8am and finally leaving at 1.20pm.

From 4pm to 6pm is 120 minutes. 120/80 = 1 time, first arriving at 2pm and finally leaving at 5.20pm.

So the barge can visit 5 times per day at spring tides.

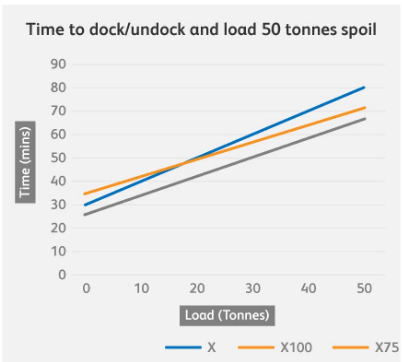
Answers - Task 4

You should use c and m for each barge as you did in task 2:

The constant c = total dock and undock time (30 minutes for barge X and 20 minutes for barge Y)

m = the time to load 10 tonnes (10 minutes for X and 15 minutes for Y).

The X75 version of the barge is most efficient – it takes only 68 minutes to load.



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DIG DEEPER

Towing multiple barges

In practice, a tug will tow several barges at once with each barge taking turn to be loaded with spoil. Can students model the total time to load and unload multiple barges, allowing for docking and undocking times and to load each barge in turn? To add interest, you could state that after the first barge, additional barges take a shorter time to dock and undock, for example half the time taken for the first barge.

Adding further barges will reduce the speed at which the tug can tow them to their destination. 30 miles downstream. For example, the tug's speed when towing one barge might be 15 mph. For each extra barge, this speed is reduced by 3mph. Can students model this to find the most efficient number of barges to tow?

Modelling other processes using y = mx + c

Students could model other examples of real life activities using graphs of the form y = mx + c. For example:

A lorry journey: the constant can represent the time to load and unload the lorry, and the gradient can represent the journey along roads of different speed limits. Can students explain how they could use these graphs to compare, for example, a journey on a 30mph limit road v a slightly longer journey on a 40mph limit road, to see which route might be most efficient?

A barge's journey to different unloading locations: students can replicate the above using barges of different speeds, for example 5 or 10mph. More able students could show how the gradient of their graph might change as the barge travels with the tide or against it, and so how a journey time might differ under these different conditions.