TUNNELWORKS
KS4 MATHS LESSON 2 (ESSENTIALS)
TEACHERS' NOTES

About this lesson

This lesson explores the geometry of circles and circular prisms (cylinders). Students must work out the number of concrete segments that line each section of tunnel, using arcs to help them. They must then calculate the number of rings that will line the tunnel, before using their knowledge of circular prisms to work out the volume and mass of concrete that will be needed.

Learning outcomes

Students can:
- Identify arcs and sectors
- Calculate the length of an arc
- Calculate the volume of a circular prism
- Work with units of length, area, volume, density and mass.

Curriculum links

KS4 Maths (may be more suitable for Higher Tier GCSE Maths students)
- Key concepts 1.1a-c; 1.2b; 1.3b; 1.4a
- Key processes 2.1a, c, d; 2.2a-c, h, j-n; 2.4a-b
- Range and content 3.2a-c

What you will need

- KS4 Maths Lesson 2 presentation
- Lesson 2 worksheet

Students are likely to need calculators for the worksheet challenges.

Preparation

Review the KS4 Maths Lesson 2, presenter's notes and the worksheet. Decide on which content you will include in your teaching, and adjust the timings below to suit your own lesson length, or to spread the content across two lessons.
Note: The word 'segments' as used in the Thames Tideway Tunnel project, refers to concrete panels that cover an arc of a circle, and not to segments in their mathematical sense.

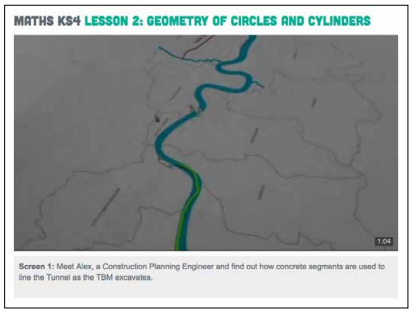
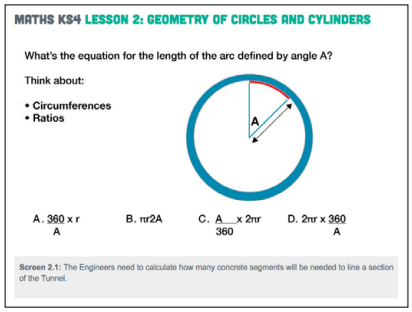
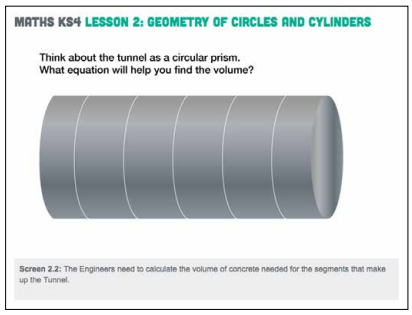
Time (60mins)	Teaching activity	Learning activity	Assessment for learning
5 mins	**Starter:** Ask students to calculate the surface area around the side of a drink can, 115mm tall and diameter 66mm.	Students make the calculation using the circumference x length. Link in discussion to the inner surface of the Tunnel.	Verbal answers, discussion.
15 mins	**Whole-class:** Optional: Show the intro video and explore the map and diagram in screens 1.1 and 1.2 of Lesson 1 if you wish. Watch the video in screen 1 and find out what students must do. Work through the interactive in screen 2.1 (not the last step) and answer each question.	Students answer onscreen questions. Discussion as per ideas in presenter's notes.	Verbal answers. Discussion, questioning.
15 mins	**Pairs/individuals:** Read the email on the worksheet and review the data students are given. Explore screen 3.1 and 3.2. Help students extract data from the diagrams and complete challenge 1.	Students use the areas given to work out the arc length for each segment, then use inner tunnel diameter to work out how many are in a ring. Share answers and calculation steps. Share strategies to calculate no. of rings and segments.	Written work. Verbal answers.
5 mins	**Whole-class:** Review students' answers and read challenge 2. Review strategies to work this out.	Students complete challenge 2 making sure their units are consistent (metres).	Discussion, questioning.
5 mins	**Individuals:** Students complete challenge 2.	Share answers.	Written work.

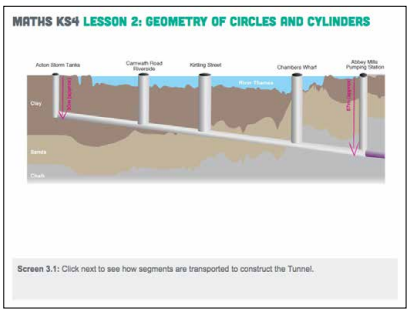
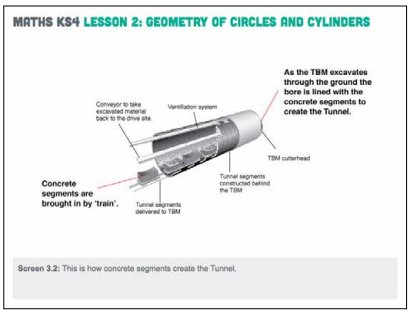
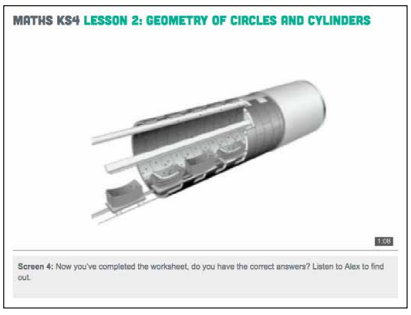
10 mins	**Plenary:** Review students' answers.	Calculate volume of concrete for whole Tunnel section using concentric cylinders.	Verbal answers. Written work.
5 mins	Guide students to calculate volume and mass of concrete required (Optional: see last step, screen 2.) Watch the answers video on screen 4.	Discuss importance of knowing volume in relation to raw materials, delivery schedule etc	Discussion.

Differentiation

Easier	Harder
Complete challenge 1 in stages and check progress on the way. Complete challenge 2 as a class discussion. Write the formula for the volume of a circular prism on the board to help with challenge 3.	Students complete individually. Don't show or discuss formulae in advance. Ask students to create the formula for arc length by considering the ratio of angles. $1m^3$ concrete requires 320kg cement, 600kg sand, 1200kg gravel and 176l water. Ask students to create a bill of materials for 100m or 1km of tunnel. Challenge students to create a spreadsheet to calculate the segments and concrete needed for any length of tunnel at any thickness, and to accommodate 3, 6 or 8 segments per ring of 1.5m length. Use the ideas in Dig Deeper.

Presentation Notes
Lesson 2

Screen	Notes
1  	Use this video to set the scene and introduce the challenge for KS4 Maths Lesson 2.
2.1  **2.2**  	Use this slide to remind or introduce students to the equation for calculating the length of an arc, shown in red. Think first how to work out the circumference of the circle and then multiply this by the ratio of the angle and a full circle e.g. A: 360. Students will need this equation to work out that there are six large segments and one small segment. They must first calculate the arc length of the two segments by thinking about its inner surface area, knowing each one's length (1.5m) and area. This gives the arc as 3.743m and 2.042m respectively and angles of 55° and 30°. Students need to think about how they would calculate the volume of a circular prism (cylinder) using πr2L where L is the length of this section of the Tunnel (5.8km) and r is the radius. The simple trick is to think of one cylinder inside the other and subtract the inner volume from the outer. Give students the inner diameter, 7.2 and the thickness of a segment - 0.35m. So the outer diameter is 7.9m. Halving each of these gives r (inner) = 3.6m and r (outer) = 3.95m. Help students identify the inner diameter using the information in the diagrams – the outer diameter of the Tunnel and the thickness of a segment.

Screen	Notes
3.1  **3.2**  	Use this diagram and the next one to set the challenge for KS4 Maths Lesson 2 and show how the tunnel is created by lining the excavated bore with concrete segments.
4  	Use this to provide feedback and answers for KS4 Science Lesson 2.

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KS4 MATHS DIG DEEPER
EXTENSION IDEAS

Grouped frequency tables

Challenge students to estimate the mean of this grouped set of data using midinterval values, and to estimate it from a histogram:

Years' experience of site crew

Years	0 - 5	6 – 10	11 - 15	16 – 20	21- 25	26 – 30	31 - 35	36 - 40	41 - 45
Freq.	24	35	58	79	116	65	37	24	12

Where are the class boundaries?

What's the mode?

In which group would the median length of experience lie?

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Cumulative frequency

Using the table above, challenge students to work out a row for cumulative frequency, using class boundaries of 5.5 years, 10.5 years etc.

What are the median, upper and lower quartile values for years of experience?

What's the inter-quartile range?

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Mean, Mode and Median

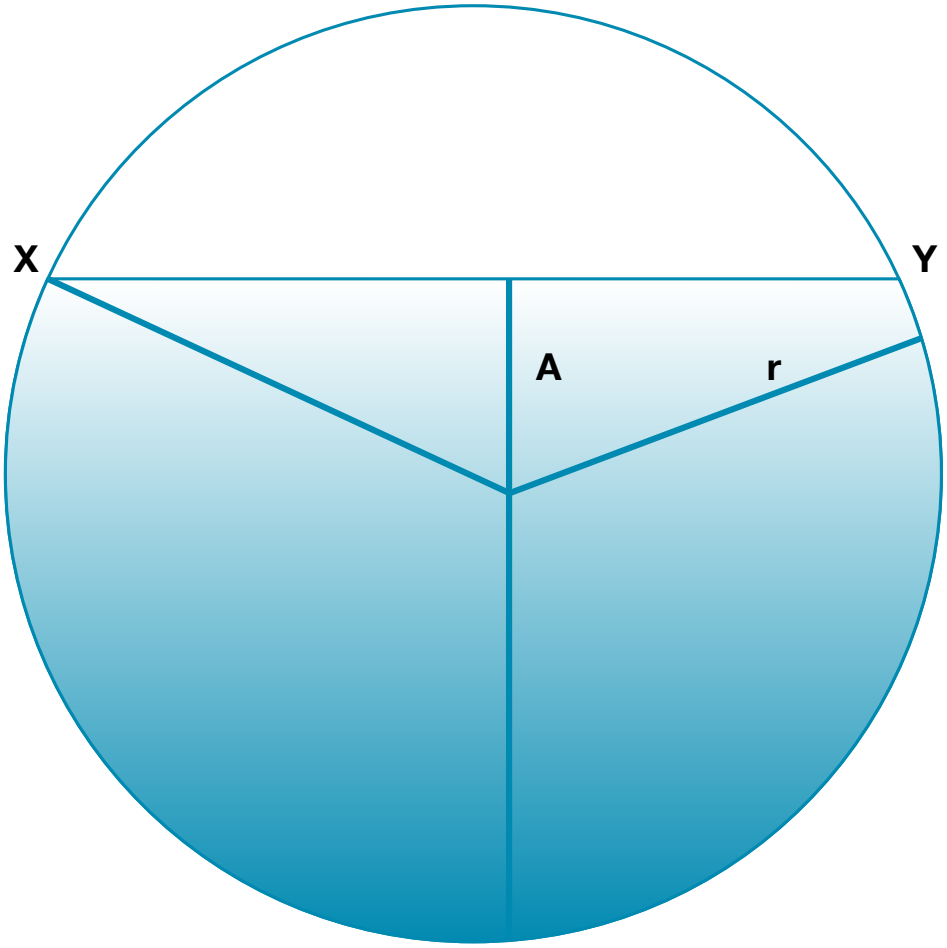
Ask students to imagine that they are the site manager in charge of ordering safety (PPE) equipment for the site, including boots and protective coats. What information might they need to gather to order the right quantities of each item?

What information is provided by the mean, mode and median of this data, and how might it be used by the manufacturers of the clothing or equipment, and the site manager?

3D thinking

The tunnel is designed to fill and capture water until it is pumped out for treatment.

In reality, the Tunnel has a gentle gradient of 1m in 790m, to ensure a clear flow. But assuming the tunnel is level, can students use their knowledge of arcs, sectors, segments and trigonometry to calculate the volumes of water stored per km in the examples below?



Remember that the inner diameter is 7.2m.

1 The angle A is 120°

2 The length X – Y across the surface of the water is 6m (this is a trick – the water could be high or low in the tunnel!)