TUNNELWORKS
AS MATHS
TEACHERS' NOTES

About this lesson

This lesson helps students practice generating and transforming trigonometric graphs, including translations and stretches. Using information about tidal patterns in the River Thames, students develop a simplified tidal chart for London Bridge before applying their understanding to modify their chart for a Thames Tideway Tunnel construction site that will be built in the foreshore of the River Thames.

Learning outcomes

Students can:

- Generate a basic graph for a trigonometric function
- Translate the curve parallel to the x and y-axes
- Apply a stretch parallel to the y-axis using a scale factor.

Curriculum links

AS maths (all specifications)

- Pure mathematics - trigonometry

What you will need

- AS Maths presentation screens AS 1- AS 4
- Student sheet

Students will need plain and / or graph paper and should ideally have access to (and the knowledge to use) an online graphing tool (eg http://www.onlinefunctiongrapher.com) or graphing calculators. Please allow time to show the introductory video at the start of the presentation.

Preparation

Students should already be familiar with trigonometric functions and basic translations.

Watch the introductory video if you are not familiar with the Thames Tideway Tunnel. Review the delivery plan for the lesson (below) and the student sheet and consider appropriate differentiation for your class. Adjust the timings to suit your lesson length or to extend to a double lesson.

Instead of using the data provided you might want to generate the initial London Bridge tidal chart using a current or recent date. It's best to use a day where the first tide is a high water. If you're confident reading tide tables, Google 'pla tides' to obtain Port of London Authority predictions for London Bridge.

Background information

The Thames Tideway Tunnel is a major new sewer that will help tackle the problem of sewage overflows from London's sewers and will protect the River Thames from increasing pollution for at least the next 100 years, enabling the UK to meet European Union environmental standards. The Tunnel will control the 34 most polluting combined sewer overflows (CSOs), as identified by the Environment Agency, which currently discharge untreated sewage directly into the River Thames after it rains.

The majority of London's sewers collect both sewage and rainwater and after heavy rainfall the volume flowing through the sewers is a lot higher than the system can take. The original Victorian system is designed to discharge this excess sewage directly into the River Thames through a series of combined sewer overflows (CSOs). The new Thames Tideway Tunnel will control the most polluting CSOs by capturing combined rainwater and sewage and allowing it to flow down a new tunnel before being pumped out for treatment at Beckton Sewage Treatment Works.

A number of the Tunnel's proposed construction sites will be built on the foreshore and 'cofferdams' will be used to enclose and retain the new structure. Material for behind the cofferdam will be loaded from barges that at low tide rest on 'campsheds' – raised platforms built next to the cofferdam. As the Thames is tidal up to Teddington Lock, the barge operators will need to know when they will be able to moor above the campshed at high water. They will therefore need accurate tidal predictions for each of these sites.

While local and regional conditions make reality more complicated than this, it's possible to represent tides using a simple trigonometric graph.

Delivery

The 'Summary of functions' after this table, will help you.

Time (60mins)	Teaching activity	Learning activity
10 mins	**Starter:** Starter: Review trigonometric graphs. Ask students to create pencil sketches of basic sine and cosine trigonometric curves and for one function, to sketch two curves that show the effects of a translation of 90° and a scale factor of 2. Screen AS 1 Screen AS 2: If you wish, also show the introductory videos about the Thames Tideway Tunnel.	Students sketch sine and cosine graphs. Students sketch curves to show a translation and scale factor. Students share equations for simple transformations.
5 mins	Discuss the idea that a trigonometric graph can represent tides in simplified form. Review the key facts and identify how each fact will help generate the features of the graph. Screen AS 3	Students relate key facts to basic graphs and transformations.
20 mins	**Whole class:** Complete task 1 to generate a graph for the tides at London Bridge. Ask students to identify which function will show high tide at time x=0. (Ans: y=cos(x)) Ask students to generate or sketch this basic graph. Discuss how to transform the graph so the difference between its min and max value is equal to the tidal range. (Ans: use MHSW – MLWS to obtain range of 6.5m then apply a scale factor of 3.25 to get y=3.25cos(x)). Discuss how to transform the graph so the total values of low and high water reflect their real values. (Ans: add CWD + MLWS to obtain min depth of 2.4m. Combine this with the graph's min value of -3.2 to obtain 5.6. Add this to the function to get y=3.25cos(x) + 5.6). Optional: Try doing this the other way around (translate along y axis first, then apply scale factor). Is the graph still accurate, showing min depth 2.4m and max depth 8.9m?	Students identify correct function and sketch the basic graph. Students use MHWS and MLWS data to calculate range and identify scale factor to adjust graph so it has the same range. They can sketch or generate a new graph. Students use CWD and MLWS data to calculate min. depth, and then calculate value of translation to shift the graph upwards so its min. value matches this. Optional: students can shift upwards first then apply scale factor. They can relate this to commutative properties in algebra.

20 mins	**Individuals or pairs:** Complete Task 2 to generate graphs for construction locations. Screen AS 4: Ask students to identify how to translate the graph to show first high tide at a different time to London Bridge. (Add a value to x). Assuming the x-axis uses degrees, how many degrees represent 1 minute? (Ans=0.5o). How many degrees represent the delay due to 1km distance upriver and should they be added or subtracted from x? (Ans: 2.5o per km, subtracted, eg x – 5o for 2km upriver.) Using this to transform their graph for tidal time delays for locations upriver, students write and generate graphs for one or more locations.	For one or more locations, students calculate the angle to subtract (or add for the first location, which is downriver) to translate their graph to show the right time delay for high tide compared to London Bridge. They then modify their function to account for different values of CWS etc, applying what they learned in task 1. Students generate graphs and can identify the time of the first high water (remembering this is at 6mn at London Bridge).
5 mins	**Whole class:** Share functions graphs for each location. Discuss how students have transformed the basic cosine function to account for time delays, water depth and tidal range. **Plenary:** Review learning. Use one or more of the extension ideas if you wish.	Students share and discuss results. Students discuss ideas and can try solving the problem.

Differentiation

Easier	Harder
Use the data to focus on one form of transformation only, eg: translations along X-axis using distance / time delays, or to obtain range.	Challenge students to generate graphs for more than one location using all data. Include one or more extension activities.