TUNNELWORKS
A2 MATHS
TEACHERS' NOTES

About this lesson

This lesson helps students apply translations and stretches to graphs of complex functions, to model real-life situations. Using data about the duration and intensity of rainfall, they modify a function to model the flow of water through a sewer and – past a particular flow threshold - through a combined sewer overflow (CSO).

Learning outcomes

Students can:
- Apply stretches to a complex function
- Apply translations to a complex function.

Curriculum links

A2 (A level) maths
- Pure mathematics - functions

What you will need

- A2 Maths presentation screens A1-A4
- Student sheet

Students may also want to use an online graphing tool (eg http://www.onlinefunctiongrapher.com) or graphing calculators. They can also enter their functions directly into the Google search bar to see a sketch of their graph. Please allow time to show the introductory video at the start of the presentation.

Preparation

Students should already be familiar with functions and translations.
Watch the introductory video if you are not familiar with the Thames Tideway Tunnel. Review the delivery plan for the lesson (below), the overview of the functions students can explore, and the student sheet. Consider appropriate differentiation for your class using the ideas after the delivery plan. Adjust the timings to suit your lesson length.

Background information

The Thames Tideway Tunnel is a major new sewer that will help tackle the problem of sewage overflows from London's sewers and will protect the River Thames from increasing pollution for at least the next 100 years, enabling the UK to meet European Union environmental standards. The Tunnel will control the 34 most polluting combined sewer overflows (CSOs), as identified by the Environment Agency, which currently discharge untreated sewage directly into the River Thames after it rains.

The majority of London's sewers collect both sewage and rainwater and after heavy rainfall the volume flowing through the sewers is a lot higher than the system can take. The original Victorian system is designed to discharge this excess sewage directly into the River Thames through a series of combined sewer overflows (CSOs). The new Thames Tideway Tunnel will control the most polluting CSOs by capturing this excess water and allowing it to flow down a new tunnel before being pumped out for treatment at Beckton Sewage Treatment Works.

When designing the Tunnel, engineers had to understand the volume of water that it may have to capture during wet weather events. One way to understand this is to create mathematical models of how storm water flows into each CSO. Doing this ensures the Tunnel's design can cope with the range of flows into it, and ensures that the Tunnel is large enough to capture and store the required amount of water until it is pumped out for treatment.

This activity uses a simple model that students can modify to suit different weather conditions and even CSO designs. In reality, the new Tunnel's design used computerised weather simulations and water flow modelling far more complex than this.

Delivery

The 'Summary of functions' after this table, will help you.

Time (60mins)	Teaching activity	Learning activity
5 mins	**Starter:** Introduce the scenario using the optional Introduction video before showing Screens A1 and A2.	Students watch the introductory video before reading Screens A1 and A2.
10 mins	**Whole class:** Use Screen A3 to introduce the basic curve. This shows the flow through a sewer during rainfall of 2mm / hour which lasts for one hour. Note that the curve doesn't represent the rain itself, but the resulting flow through the sewer, which last for two hours in total. Establish what the area under the curve represents. Establish that the green line shows the sewer's maximum flow capacity. That is, the flow rate beyond which any further flow will exceed the capacity of the sewer and overflow into a CSO which currently discharges directly into the River Thames. Ask what happens when the flow is more or less than this (eg part of or none of graph lies above this line). Ask students how rainfall of different intensities and durations would affect the flow through the sewer and what transformations to the graph of this basic function could represent these. Gather ideas. Ask students to sketch two transformations, one for a rainfall event of twice the intensity (eg 4mm / hour), and one for an event of half the duration (a 30 minute shower that causes water to flow through the sewer for an hour in total). Invite students to share their sketches then show Screen A4.	Review basic transformations (stretches and translations) along x and y for a simple function y = f(x). Sketch transformations.
10 mins	**Individuals / pairs:** Review Task 1, which students should then complete. (*Hints: students should remember that their curves will still cross the X-axis where x=0 and x=2 hours, and use the given values for maximum flow as their new maximum value for y.*)	Sketch transformations and shade areas above $0.6m^3/s$ level.

10 mins	**Whole class:** Review students' sketched and shaded curves. Show Screen A5. This shows a function that describes the 'basic' rainfall event that's just on the sewer's maximum capacity/ CSO threshold. Review Task 2. Discuss how students can modify the function to complete the transformations. If you wish, first use the examples from Task 1 to work through as a whole class (eg for rainfall of 9mm / hour, the scale factor to apply to the equation is 0.98/0.6 = 1.633 f(x).)	Identify scale factors and how to add these to the equations.
10 mins	**Individuals:** Students complete Task 2. (*Hints: The only important information is the maximum flow rate or the total time for which water flows. The actual rainfall intensity or duration aren't required.*) To complete the final two transformations students need to remember that the scale factor for a stretch parallel to the x-axis must be inverted. (Eg for rainfall half as long as the 'basic' rainfall event, the scale factor must be 2 (eg f(2x).)	Identify scale factors and how to add these to the equations.
10 mins	**Whole class:** Review students' modified equations. Show Screen A6. Explain that because homes and businesses continually generate wastewater there is always a minimum flow of $0.4m^3/s$ through the sewer even when there is no rain. Review Task 3. Establish that students need to think about the 'basic' rainfall event again: 2mm / hour, which creates a maximum flow rate of $0.6m^3/s$ through the sewer. **Individuals:** Students complete Task 3, identifying a compression parallel to the y-axis with scale factor 1/3, and a translation of +0.4 parallel to the y-axis, as the two transformations required.	Identify scale factor and translation and how to add these to the equation.
5 mins	**Whole class:** Review students' modified equations. (The equation becomes f(x) = 0.1666x3 – 0.6666x2 + 0.6666x + 0.4). Discuss the value of modelling the real world using maths.	

Differentiation

Easier	Harder
Limit the number of transformations students must complete, and omit Task 3.	Ask students to plot or sketch each curve and estimate for how long the sewer capacity is exceeded and the CSO might currently discharge into the River Thames (e.g. the time for which the flow is > $0.6 m^3/s$ in each case). Challenge students to combine transformations using the tables below, to find the equation that describes rain events of different intensities and durations. Challenge students to sketch and write the equations that represent a delay between the start of the rain event and the first flow through the sewer (you could ignore the minimum flow from Task 3 here), for delays of 15 and 30 minutes. Again students could combine these with other transformations. Some students could integrate one or more curves, remembering to convert from hours to seconds, and find the total flow during the rainfall and total discharge above $0.6 m^3/s$.

Data for harder activities

Rainfall intensity (mm / hour)	Maximum flow in sewer
0	0.40
1	0.50
2	0.60
3	0.65
4	0.70
5	0.76
6	0.82
7	0.87
8	0.93
9	0.98
10	1.04
11	1.10
12	1.14
13	1.17
14	1.20
15	1.21
16	1.22

Rainfall duration (hours)	Sewer flow duration (hours)
0.5	1.0
1	2.0
1.5	2.5
2	3.0
2.5	3.5
3	4