TUNNELWORKS
KS3 MATHS LESSON 1 (ESSENTIALS)
TEACHERS’ NOTES

About this lesson

This lesson explores right-angled triangles and Pythagoras’ Theorem using simplified engineering drawings. Students use their knowledge to calculate missing dimensions from two drawings, before calculating the gradients of some sections.

Learning outcomes

Students can:

- Name and recall Pythagoras’ Theorem
- Transpose the formula to work out the hypotenuse other sides
- Apply the formula to calculate missing dimensions
- Calculate and express a gradient as a fraction or decimal.

Curriculum links

KS3 Maths

- Key concepts 1.1a-c; 1.2b; 1.4a
- Key processes 2.1a, c, d; 2.2a-c, h, j-n; 2.4a-b
- Range and content 3.2a, c, f

What you will need

- KS3 Maths Lesson 1 presentation
- Lesson 1 worksheet

Students are likely to need calculators for the worksheet challenges.

Preparation

Review the KS3 Maths Lesson 1 presentation and presenter’s notes, and the student worksheet. Decide on which content you will include in your teaching, and adjust the timings below to suit your own lesson length, or to spread the content across two lessons.

Time (60mins)	Teaching activity	Learning activity	Assessment for learning
5 mins	**Starter:** Students share their ideas about types of triangles and their properties, in pairs and then discuss as a class.	Students share ideas.	Discussion, questioning.
15 mins	**Whole-class:** Show the intro video and explore the map and site in screens 1.1 - 1.3. Watch the video in screen 2 to find out what students must do. Work through the first two parts of the interactive in screens 3.1 and 3.2 and answer each question.	Students suggest real-life examples of triangles around them in school or home. Answer on-screen questions.	Discussion, questioning. Verbal answers.
15 mins	**Individuals or pairs:** Read the email on the worksheet and review the diagrams. Students complete challenge 1 on the worksheet, remembering to work to 3d.p.	Students calculate the missing dimensions. Share answers and calculations.	Written work.
5 mins	**Whole-class:** (Optional) review students' answers before starting challenge 2. Review the last part of the interactive on screens 3.3 - 3.5.	Link ideas to gradients of roads etc.	Written work.
5 mins	**Individuals or pairs:** Students complete challenge 2. **Plenary:** Review students' answers and watch the answers video on screen 4.	Students calculate the gradients. Students discuss why the gradients might be important to get right and suggest where else gradients can matter (e.g. hills for vehicles).	Discussion, questioning.

Differentiation

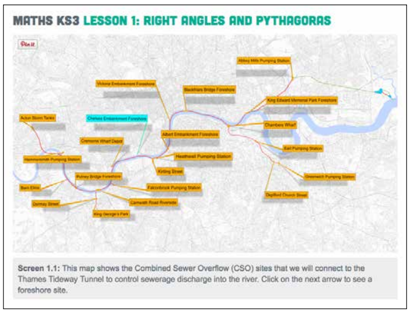
Easier	Harder
Practice transposing the formula to find each side. Complete a couple of easier examples as a whole-class activity, one without decimals and one including decimals (eg find the hypotenuse for triangles with sides 14m and 10m, and 10.450m and 5.250m). Omit some or all of challenge 2.	Challenge students to make a geometrical proof of the theorem using squared paper. See if students can make a 3-4-5 triangle into two smaller right-angled triangles and explore how they relate. Try using each side as the radius of a circle and use the formula for the area of a circle instead of a square –it still works! Explore different gradients for roads, eg: 1 in 5 to 1 in 3. Try expressing in different ways and drawing the slopes.

Presentation Notes
Lesson 1

Screen	Notes
Intro 	This short video introduces the Thames Tideway Tunnel. It is just under 3 minutes long. Use the video at the start of KS3 Science Lesson 1 (and Lesson 2 as well is you wish) to set the overall scene for your students. **About the Thames Tideway Tunnel:** 39 million tonnes of untreated sewage overflows into the River Thames each year from London's Victorian sewerage system. The Thames Tideway Tunnel is a major new sewer that will tackle this problem, protect the River Thames from increasing pollution for at least the next 100 years, and enable the UK to meet European environmental standards. Though built to last and in good condition, the existing sewerage network is now too small to transfer all London's sewage to our treatment works for processing (after rainfall). London's sewerage system dates from the 19th Century and was designed as a combined system. This means that a single pipe carries both foul water (from homes and businesses) and rainwater run-off (from streets, roofs and parks) to sewage works for processing before being discharged into the River Thames. Increasingly, when it rains in London there is not enough capacity in the sewerage network to convey all the rainwater as well as foul flows. The system was designed to overflow into the River Thames so that peoples' homes and streets are not flooded with untreated sewage. The system does this through combined sewer overflows (CSOs) on the banks of the River Thames. Thames Water have worked with the Environment Agency to identify the most polluting CSOs – the ones that cause unacceptable environmental impacts because of the frequency or volume of the overflow, or because they discharge into an environmentally sensitive part of the river. The Thames Tideway Tunnel Project will address the overflows from these CSOs, either by directly connecting them to the tunnel, or by making other alterations to the sewerage system which will utilise the existing capacity more effectively. The flows diverted into the Thames Tideway Tunnel will be stored in the tunnel and pumped out for treatment at Beckton Sewage Treatment Works in east London. The CSOs will still be needed after the Thames Tideway Tunnel has been built to direct flows to the River Thames in exceptional circumstances when the new tunnel system is full. This is only expected to occur very occasionally. The Thames Tideway Tunnel will also bring wider social and economic benefits: A cleaner, healthier River Thames is essential for the wellbeing of the city as a whole. There will be less pollution and more dissolved oxygen in the water. The Thames Tideway Tunnel will ensure the country's capital remains a flourishing business centre and tourist destination, protecting the city's reputation around the world. The river is a great, under-used asset for the capital that must be protected.

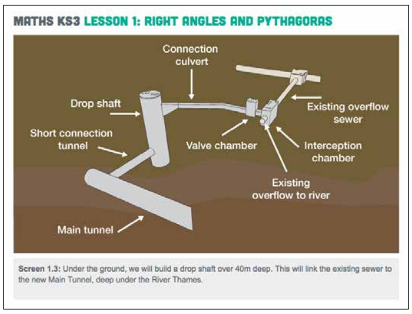
Screen	Notes
1.1, 1.2, 1.3	Use these three screens to set the scene for lessons 1 and 2 in KS3 Maths. Victoria Embankment is a typical CSO site, where they will link an existing sewer to the new Thames Tideway Tunnel. Under all but the most exceptional conditions, the modified CSOs will divert rainwater and sewage down into the Tunnel rather than letting it discharge into the river Thames. At the moment, the outfall at Victoria Embankment discharges about 94,000 m3 into the river each year. That's about as much as in 38 Olympic-sized swimming pools. This is a good example of why building the Thames Tideway Tunnel is so important. The drop shaft takes the diverted water and sewage down to the same level as the main tunnel. It's important to design each part of the CSO correctly so water will flow easily through it, but not so fast that the flow may damage the new structure.
2	Use this video to set the scene for KS3 Maths Lesson 1 and introduce the challenge to students.

1.1



1.2

1.3



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<tr><th>Screen</th><th>Notes</th></tr>
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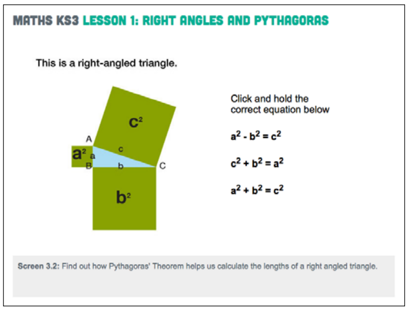
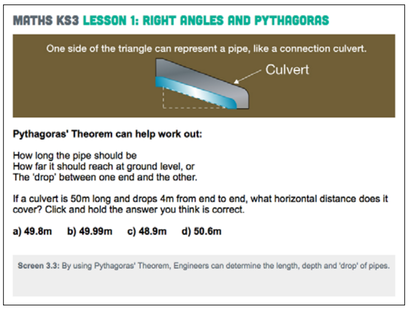
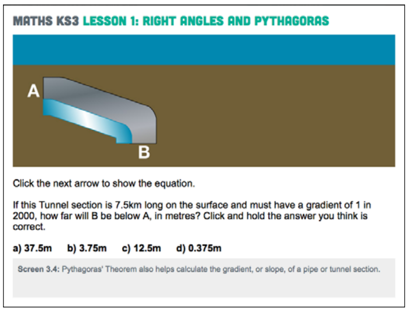
3.1



3.2



3.3



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Screen 3.1 introduces Pythagoras' Theorem. Pythagoras lived about 2500 years ago, but it's not certain that he was actually the inventor: the Babylonians, Egyptians and ancient Indian civilisations all appear to have known of the relationship within a 3-4-5 triangle.

Click the arrow to reveal the squares on each side of the triangle ask students to choose the correct equation. Ask students to try it themselves using a 3-4-5 triangle.

Screens 3.3 - 3.5 show some practical uses for Pythagoras' Theorem in engineering: finding lengths, the horizontal distance covered at ground level (make sure students are clear on what this means) and the 'drop' between one end and the other, which is important for ensuring correct flow along the pipe or tunnel.

Even if engineers aren't doing this by hand, the computer-aided design (CAD) software they use does this in order to render 2D and 3D drawings, plans and elevations.

For example, a pipe linking two structures will not be the same length as their distance apart, because of the 'drop' or gradient of the pipe. Here, the solution is the square root of 502 – 42.

See if students can think of other places where Pythagoras can help engineers draw up plans. This could include sloping entrances to buildings, sloping roofs.

In reality, engineers often think about gradients when designing a network of pipes and tunnels like in the Thames Tideway Tunnel. Knowing the gradient that must achieve, they can calculate the necessary drop between different points along the tunnel, for example between each CSO as the tunnel makes its way from West to East.

A gradient can be thought of as a fraction (e.g. 1 in 100) or as its decimal equivalent (which is what engineers use and what students will use in the final part of the lesson).

1 in 2000 = 0.0005, and 0.0005 x 7500 = 3.75m
NOTE: THIS IS NOT THE REAL GRADIENT OF THE MAIN THAMES TIDEWAY TUNNEL, WHICH STUDENTS WILL WORK OUT IN THE LAST PART OF THE LESSON.

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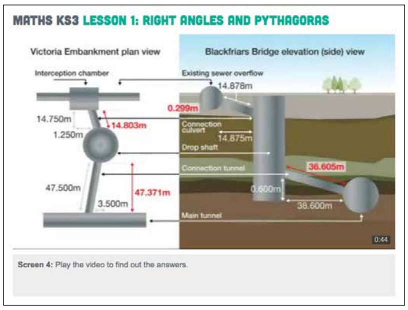

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Use this to provide feedback and answers for KS3 Maths Lesson 1.

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