

TUNNELWORKS

KS4 MATHS LESSON 1 (ESSENTIALS)

TEACHERS' NOTES

About this lesson

This lesson explores mean, mode and median. It uses fictitious data on the times between unplanned maintenance for up to four different parts on a Tunnel Boring Machine (TBM). Students create a frequency table to organise their data before calculating the mean time between events and using this information to plan an inspection schedule.

Learning outcomes

Students can:

- Explain the difference between mean, mode and median
- Organise data into a frequency table
- Calculate the mean from a frequency table
- Consider the real-world meaning and importance of mean values.

Curriculum links

KS4 Maths (also suitable for all GCSE Maths students)

- Key concepts 1.1a-c; 1.2b; 1.3b; 1.4a
- Key processes 2.1a, c, d; 2.2a-c, h, j-n; 2.4a-b
- Range and content 3.3a-c

What you will need

- KS4 Maths Lesson 1 presentation
- Lesson 1 worksheet

Students are likely to need calculators for the worksheet challenges.

Preparation

Review the KS4 Maths Lesson 1 presentation, presenter's notes and worksheet. Decide on which content you will include in your teaching, and adjust the timings below to suit your own lesson length, or to spread the content across two lessons.

Consider ideas to include that explore more examples of gathering data, calculating means and using this information in the real world, such as age, height, waistline, income etc.

Time (60mins)	Teaching activity	Learning activity	Assessment for learning
5 mins	Starter: Challenge students to work out the average on each table, in days.	Students work in groups to calculate, share answers and how they did it.	Verbal answers, questioning, discussion.
15 mins	Whole-class: To begin with, you may wish to show the intro video about the Thames Tideway Tunnel project if the class have not seen it before, otherwise start with screen 1. Explore the map and TBM diagram in screens 1.1 and 1.2. Watch the video in screen 2 to find out what students must do. Work through the interactive in screens 3.1 to 3.5 and answer each question. Use the presenter's notes ideas to extend your discussion.	Students review mean, mode, median and answer on-screen questions. Discussion as appropriate. Students complete one frequency table and calculate mean, in groups.	Verbal answers, questioning, discussion.
10 mins	Groups: Read the email on the worksheet and review the data and blank tables. Students complete the first table in challenge 1, working in small groups to organise data and calculate mean.	Share answers. Students complete two frequency tables and calculate means, in pairs or alone.	Verbal answers. Written work.
20 mins	Whole-class: (Optional) review students' answers so far.	Share answers.	Verbal answers.
15 mins	Individuals or pairs: Students complete remaining two tables and calculations in challenge 1.	Share answers.	Written work, verbal answers.
5 mins	Whole-class: (Optional) review students' answers before starting challenge 2. Plenary: Students write their email follow-up and share answers. Review students' answers. Watch the answers video on screen 4. Discuss the importance of inspection schedules and how data can contribute to safety and efficiency on site.	Contribute to discussion	Discussion.

Differentiation

Easier	Harder
Use 1 – 4 sets of data as appropriate for each student / group. Complete a worked example.	Challenge students to draw bar charts/ histograms for their data and estimate the mean. How do their estimates compare to their calculations? Explore the dispersion for each set of data. Which set has the most, and which the least? What does this say for the reliability of the data and its contribution to effective inspection schedules? How could this data be improved (e.g. more reliable)? (Add more data.) Discuss how this information could help order sufficient spare parts without holding too much stock on site.

Presentation Notes

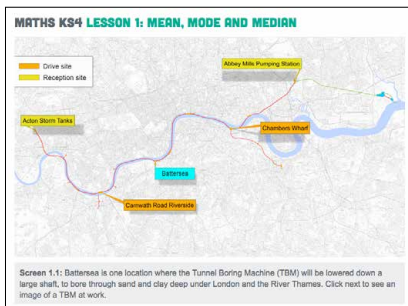
Lesson 1

Screen	Notes
Intro INTRODUCING THE THAMES TIDEWAY TUNNEL  Watch the video to find out why we need to clean up the River Thames.	This short video introduces the Thames Tideway Tunnel. It is just under 3 minutes long. Use the video at the start of KS3 Science Lesson 1 (and Lesson 2 as well if you wish) to set the overall scene for your students. About the Thames Tideway Tunnel: 39 million tonnes of untreated sewage overflows into the River Thames each year from London's Victorian sewerage system. The Thames Tideway Tunnel is a major new sewer that will tackle this problem, protect the River Thames from increasing pollution for at least the next 100 years, and enable the UK to meet European environmental standards. Though built to last and in good condition, the existing sewerage network is now too small to transfer all London's sewage to our treatment works for processing (after rainfall). London's sewerage system dates from the 19th Century and was designed as a combined system. This means that a single pipe carries both foul water (from homes and businesses) and rainwater run-off (from streets, roofs and parks) to sewage works for processing before being discharged into the River Thames. Increasingly, when it rains in London there is not enough capacity in the sewerage network to convey all the rainwater as well as foul flows. The system was designed to overflow into the River Thames so that peoples' homes and streets are not flooded with untreated sewage. The system does this through combined sewer overflows (CSOs) on the banks of the River Thames. Thames Water have worked with the Environment Agency to identify the most polluting CSOs – the ones that cause unacceptable environmental impacts because of the frequency or volume of the overflow, or because they discharge into an environmentally sensitive part of the river. The Thames Tideway Tunnel Project will address the overflows from these CSOs, either by directly connecting them to the tunnel, or by making other alterations to the sewerage system which will utilise the existing capacity more effectively. The flows diverted into the Thames Tideway Tunnel will be stored in the tunnel and pumped out for treatment at Beckton Sewage Treatment Works in east London. The CSOs will still be needed after the Thames Tideway Tunnel has been built to direct flows to the River Thames in exceptional circumstances when the new tunnel system is full. This is only expected to occur very occasionally. The Thames Tideway Tunnel will also bring wider social and economic benefits: A cleaner, healthier River Thames is essential for the wellbeing of the city as a whole. There will be less pollution and more dissolved oxygen in the water. The Thames Tideway Tunnel will ensure the country's capital remains a flourishing business centre and tourist destination, protecting the city's reputation around the world. The river is a great, under-used asset for the capital that must be protected.

Screen

Notes

1.1

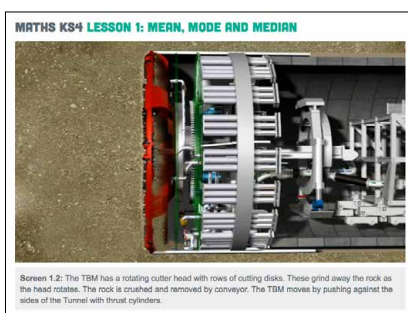


Use this map to set the scene. Battersea will be the location of a 'main drive site': one of a few where the Tunnel Boring Machine (TBM) will be lowered deep under London. The TBM weighs about 800 tonnes, so it will be lowered bit by bit and assembled at the bottom of the shaft. An incredibly accurate positioning and control system will guide the TBM along the tunnel route until it emerges at the next reception site, where it will be raised in parts back to the surface.

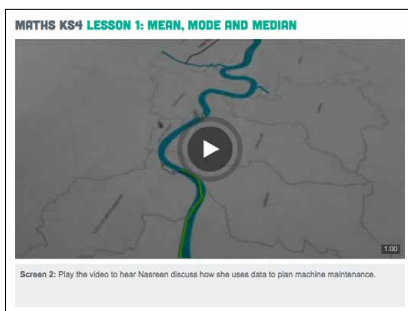
TBMs use a rotating cutter head to grind away rock. These contain rows of hardened metal disks that press against the rock and fracture pieces from the surface. These pieces fall to the bottom where they go through a rock crusher. Some types of rock can be removed in a dry state using a long conveyor that will stretch all the way back to the main shaft, where the rocks will be raised to the surface and transferred where possible to barges for removal (this reduces traffic congestion). Other rocks, like the chalk that the TBM will bore through elsewhere on the Tunnel's route, will be mixed with water to form a slurry. This will be piped back to the main shaft and up to the surface where it again will be transported away, by barge where possible.

The TBM moves by pressing forwards against the sides of the tunnel. It does this using a series of Thrust Cylinders. These are attached to hydraulic rams that press outwards against the sides. Once the TBM has moved as far forward as it can, these rams are withdrawn and repositioned further along, for the thrust cylinders to push against once again.

1.2



2



Show this video to set the scene for KS4 Maths Lesson 1 and introduce the challenge to students.

3.1



Use this picture to show students a TBM conveyor. This works at high speed, since the TBM generates a lot of waste rock. Discuss what might go wrong.

Ask students what information engineers might need to know in order to predict any failure rate and therefore plan maintenance and inspections that can keep the conveyor working and in good order. This might include the type of rock that the TBM will be boring into, since different rocks have generate different levels of dust and are of different hardness. It might also include historical data on how often a conveyor system had to be maintained under similar conditions, and this is what students are going to look at over the next few stages.

Screen

3.2

MATHS KS4 LESSON 1: MEAN, MODE AND MEDIAN



This raw data shows the number of days between conveyor shut-downs during the manufacturer's testing of a TBM:

5, 10, 11, 6, 7, 9, 12, 14, 10, 15, 8, 13, 12, 11, 7, 8, 10, 10, 9, 11, 9, 10

The engineers need to know the average time before the conveyor may experience a problem so they can develop a planned maintenance schedule. Is this time the:

a) Mode b) Mean c) Median

Screen 3.2: Engineers on the Thames Tideway Tunnel often have to make sense of raw data.

3.3

MATHS KS4 LESSON 1: MEAN, MODE AND MEDIAN

To make sense of the raw data, the engineers organise it into a frequency table.

5, 10, 11, 6, 7, 9, 12, 14, 10, 15, 8, 13, 12, 11, 7, 8, 10, 10, 9, 11, 9, 10

Value	5	6	7	8	9	10	11	12	13	14	15
Frequency	1	1	2	2	3	5	3	2	1	1	1

What's the next step to calculate the mean for this data?

a) 1st row total x 2nd row total
b) Calculate value x frequency for each column
c) Find the most common

Screen 3.3: Look at the newly organised data and calculate the mean.

3.4

MATHS KS4 LESSON 1: MEAN, MODE AND MEDIAN

The engineers have created a third row showing the value x frequency:

Value	5	6	7	8	9	10	11	12	13	14	15
Frequency	1	1	2	2	3	5	3	2	1	1	1
V x F	5	6	14	16	27	50	33	24	13	14	15

What is the mean time between unplanned conveyor maintenance?

a) 9.86 days b) 7.38 days c) 10.12 days d) 11.15 days

Screen 3.4: Engineers use data to calculate the time between any unplanned maintenance.

Notes

Ask students what they would want to know in order to trust these data: that they come from the same make and model of TBM conveyor and were gathered under similar conditions, rather than, say, granite rock, which is not present under London.

Use this slide to discuss the three options:

The mode is the most common time (the most common value in the data). The mean is the average time between a need to shut down the conveyor (the sum of all the times divided by the number of occasions). The median is the value that separates the lower half of the data from the top half, found by putting the data in order and finding the middle one.

It's worth discussing why the engineers might be most interested in the mean rather than the mode:

The mean is the best way to predict future values. If you are using this lesson with more able students who will do the additional tasks in 'differentiation' (see the lesson plan), discuss how the accuracy of this prediction depends on the spread of data around the mean and whether or not there are any unusual or extreme data that do not fit the overall pattern. Discuss dispersion and range.

The mode is most useful when it's important to identify the most common or popular choice or interval, such as the popularity of different sizes of safety clothing for site workers, or even for biscuits in the site office. The median may be useful if there are extreme values in the data, because it is less affected by these than the mode.

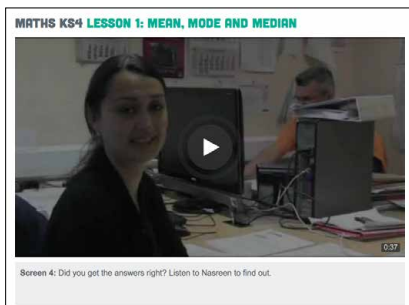
This slide shows how to organise the data into a table. Note that students need to scan the data and identify the range before they start their table. If you want students to create a sample histogram, this will enable them. Click on the next arrow to see how the data should be organised.

Students should be aware that they should not calculate the totals for each row as this doesn't take into account the frequency of each data value. They need to multiply the value x frequency for each column.

This slide now shows the calculations of value x frequency. To find the mean, students should divide the total for row 3 (the total of all the data) by the total for row 2 (the total number of data). The point of an inspection is to anticipate and prevent unexpected failure, not respond to it, so inspections should occur prior to the mean time between failures. 9 days seems about right. 10 or more days will be too late on many occasions and 9 3/4 will be cutting it too fine. They are also odd time periods to schedule on a busy site.

Screen

4



Notes

Use this to provide feedback and answers to KS4 Maths, Lesson 1.